# Chapter 15
## Electrical Potential Chapter Review

EQUATIONS:

- $V_A = \left(\frac{U}{q}\right)_A$ [This is the definition of the absolute electrical potential $V_A$. Essentially a modified potential energy function, it is a scalar quantity that identifies the amount of potential energy per unit charge available at a point. Its units are joules per coulomb, or volts, and sometimes it is called the electrical potential or just the voltage. An electrical potential field can exist whether there is a charge in the region to experience the field or not.]

- $U_A = qV_A$ [A manipulation of the definition of electrical potential, this expression allows you to determine the amount of potential energy U a charge q has when placed in a field at a point where the electrical potential $V_a$ is known. This expression is ALWAYS TRUE.]

- $\frac{W}{q} = -\Delta V$ [The work done by a conservative force as a body moves from one point to another in the force field is equal to the change of the body's potential energy in so moving, or $W = -\Delta U$. As electrical potentials are modified potential energy functions, and electric fields are modified force fields, it is true that the work per unit charge W/q available between two points in an electric field equals $-\Delta V$.]

- $W = -q\Delta V$ [This is a modification of the expression presented directly above. ALWAYS true, it relates the amount of work an electric field does on an object that moves from one point in the field to another.]

- $\boldsymbol{E} \bullet \boldsymbol{d} = -\Delta V$ [Assuming that E is constant, this is the relationship between the electrical potential difference $\Delta V$ between two points a distance d units apart, and the electric field E that must exist if the electrical potential difference $\Delta V$ is to exist. Please note that the d vector extends from the initial point to the final point, and that $\theta$ is the angle between d (as defined) and the electric field vector E. Also note that, when expanded, this dot product becomes $Ed\cos\theta = -(V_{final\ pt.} - V_{initial\ pt.})$.]

- $\int_{pt\ 1}^{pt\ 2} \boldsymbol{E} \bullet d\boldsymbol{r} = -(V_2 - V_1)$ [This integral calculates the amount of work per unit charge available between two points in a variable electric field. This work per unit charge quantity is equal to the opposite of the electrical potential difference (i.e., $-\Delta V$) between the two points. This expression is ALWAYS TRUE, but $\boldsymbol{E} \bullet \boldsymbol{d} = -\Delta V$ is easier to use if you are lucky enough to be working with a constant electric field.]

- $V(x) - V(where\ \boldsymbol{E}\ is\ zero) = -\int_{where\ \boldsymbol{E}\ is\ zero}^{x} \boldsymbol{E} \bullet d\boldsymbol{r}$ [Just as $U(x) - U(where\ \boldsymbol{F}\ is\ zero) = -\int_{where\ \boldsymbol{F}\ is\ zero}^{x} \boldsymbol{F} \bullet d\boldsymbol{r}$ is used to derive a potential energy function from a known force function, this relationship is used to derive an electrical potential function from a known electric field function. The integral determines the amount of work per unit charge done by the field as a test charge is moved from the field's zero point to some arbitrary point x. That is related to the electrical potential function as shown. Note that the notation used above is Cartesian. Though you will never use polar spherical notation, the expression is usually presented theoretically as $V(r) - V(\text{where } \mathbf{E} \text{ is zero}) = -\int_{\text{where } \mathbf{E} \text{ is zero}}^{r} \mathbf{E} \bullet d\mathbf{r}$.]

- $V(r) = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r}$ [This derived function defines the electrical potential field due to a POINT CHARGE. Use it ONLY when the field you are working with has been produced by a point charge!]

- $dV = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r}$ [This is the differential electrical potential dV generated by a differential bit of charge dq (a point charge) that is a distance r units from a point of interest. If you are dealing with an extended charge configuration and dq is arbitrarily positioned within that structure, the sum of all such dV's will yield the net electrical potential at the point. Mathematically, this is expressed as $V = \int dV = \frac{1}{4\pi\varepsilon_o}\int \frac{dq}{r}$. This approach is usually used for charged rods, hoops, and disks. It is not used when dealing with three dimensional structures like charged spheres or cylinders. There is another approach used in those situations.]

- V(r) = [V(c) - V(at zero-potential position)] + [V(b) - V(c)] + [V(r) - V(b)] [This kind of expression is used to determine the electrical potential a distance r units from a complex spherical or cylindrical charge configuration in which different electric field functions exist for different regions between the point and the field's zero point (this is often at infinity). To execute this expression, one must determine, then sum, the electrical potential differences that exist between the boundaries of those regions. To do so, Gauss's Law must be used to determine the electric field in each region, then the general relationship $[V(b) - V(a)] = -\int_a^b \mathbf{E_1} \bullet d\mathbf{r}$ must be used to determine the electrical potential jump associated with each region between the zero position and the point of interest.]

- $\mathbf{E} = -\nabla V$ [Given an electrical potential function V (remember, V is a scalar), the electrical potential's electric field E is equal to the opposite of the del operator acting on V. This suggests a clever way of determining an electric field vector. For a given charge configuration, determine the electrical potential field function AT AN ARBITRARY POINT IN SPACE, use the del operator to derive the electric field function at that arbitrary point, then evaluate that electric field function at the point of interest.]

<u>COMMENTS, HINTS, and THINGS to be aware of</u>:

- There are all sorts of things you should be able to do in this chapter. You should be able to determine the amount of work an electrical potential field does as a body moves from one point to another in the field (use $W = -q\Delta V$); relate an electrical potential difference $\Delta V$

to E in a constant electric field (use $\boldsymbol{E} \bullet \boldsymbol{d} = -\Delta V$); deal with conservation of energy when electric fields are present (remember, U = $qV_A$); derive an electrical potential function given an electric field (use $V(r) - V(where\ \boldsymbol{E}\ is\ zero) = -\int_{where\ \boldsymbol{E}\ is\ zero}^{r} \boldsymbol{E} \bullet d\boldsymbol{r}$); do this last operation when there are several unknown electric fields involved in an oddball charge configuration (a layered sphere, for instance--you'll need to utilize Gauss's Law in such cases); derive an electric field function given its associated electrical potential function (use $\boldsymbol{E} = -\nabla V$); and deal with point charges. In short, there are lots of odds and ends to deal with in this chapter.

- Remember that the electrical potential of a positive charge configuration is positive and that of a negative charge configuration is negative. Remember also that electrical potentials are SCALARS, so positive and negative signs don't denote direction.

- Note that as the del operator is a partial derivative with respect to a spatial coordinate (example: $\frac{\partial}{\partial x}$), and as $\nabla V = -\boldsymbol{E}$, the electric field is related to the rate of change of the electrical potential with position.

- You now have three ways to determine an electric field: the definition approach that defines a dq, determines dE at the point of interest, breaks dE into components, then sums via integration all of the differential fields along a particular direction to get the net field in that direction; Gauss's Law, which is useful whenever you have three dimensional symmetry; and, now, the electrical potential approach that makes use of the relationship $\boldsymbol{E} = -\nabla V$.

- Be careful when you determine a charge's potential energy in a conservation of energy problem. The potential energy of a negative charge is U = (-q)V.

- Remember, a positive charge will accelerate from higher to lower electrical potential, or along the electric field lines. Negative charges do the exact opposite, accelerating from lower to higher electrical potential against the electric field lines.

- An equipotential line is a line upon which the electrical potential is the same everywhere. Equipotential lines are ALWAYS perpendicular to electric field lines.